Analyzing a Relaxation Oscillator

<picture of real circuit>

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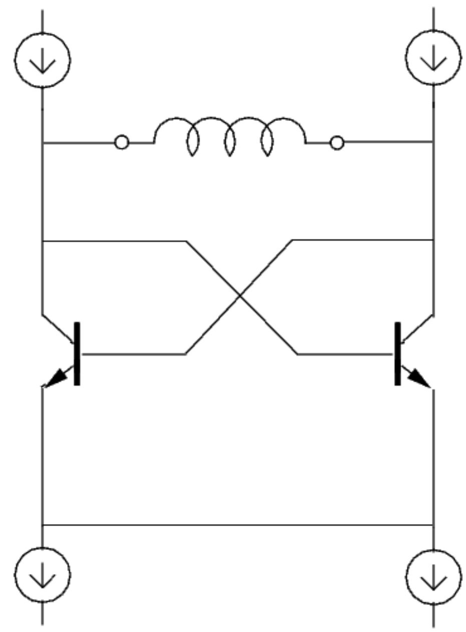
Content

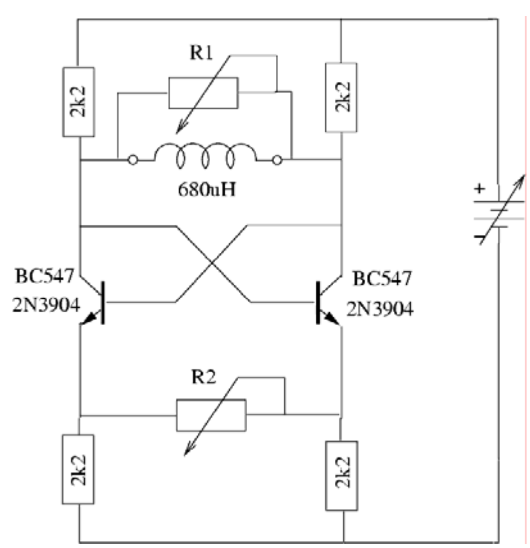
1. Introduction



In our electronics class we had to built and analyze a relaxation oscillator. A relaxation oscillator consists of two basic parts. One is a passive element that can store energy such as an inductor or a capacitor, and the other an active element with hysteresis.
The basic part of the circuit we had to build is pictured to the right. As can be seen, the passive element consists of an inductor and the active element of two balanced transistors. The inductor value and the supply voltage were 680µH and 5V respectively and we were not allowed to change these. Other requirements were that the frequency must be around 50 kHz and that the power consumption must be minimal.
Tools are not always effective when analyzing a circuit. Sometimes building and measuring can be just as effective. This was the case for us, and we only used the tools for learning purposes and to confirm that the manual analysis without tools was correct.

2. Analyzing the circuit by building it

To build the circuit, we had to construct the current sources by a voltage source and some resistors. The voltage fluctuations caused by the transistors are small in relation to the power supply. So the voltage over the resistors hardly changes, and thus the current is almost constant. By changing the voltage supply we could change the "virtual" current sources.
After building the circuit and measuring the frequency we saw that a larger current source gave a lower frequency. To reach the frequency of 50 kHz we needed a current source of 10 mA. To decrease the current source, and thus following the requirement of the minimal power consumption, we put in some variable resistors to see what effect they would have. The measurements showed that R1 decreased the frequency. We decided to not use R2 since it only had a negative effect on our circuit. From what we could see was there a specific R1 value for which frequency was minimal. Unfortunately R1 had a negative effect on the waveform, making it look less like a square wave.

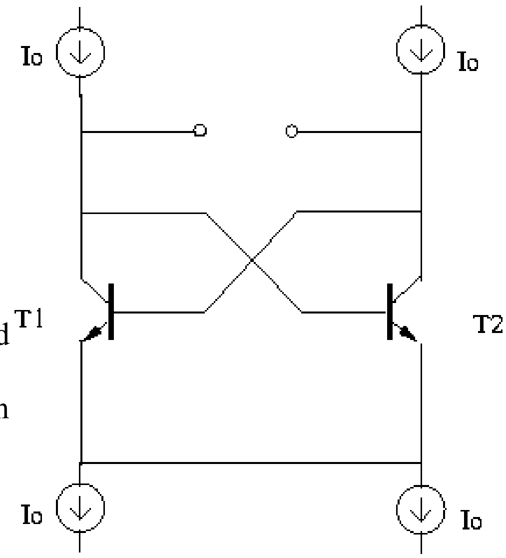


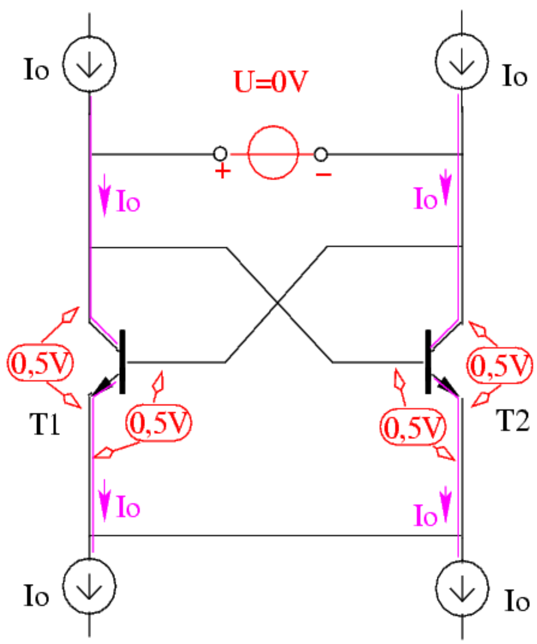
3. Analyzing the circuit conceptually

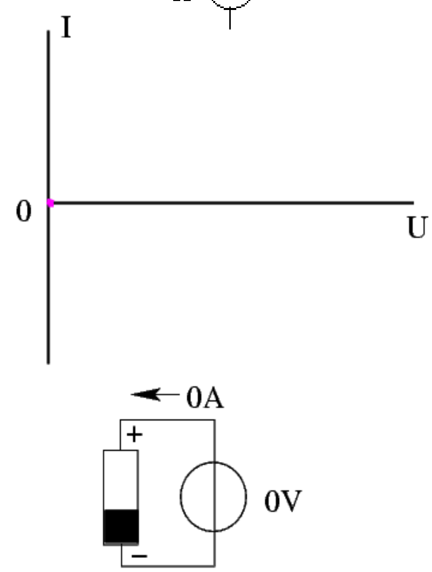
In this section we describe how the circuit can be analyzed using physical concepts and physical reasoning. Note that all voltages and currents mentioned have fictional values.

3.1 Analyzing U-I and I-U plots

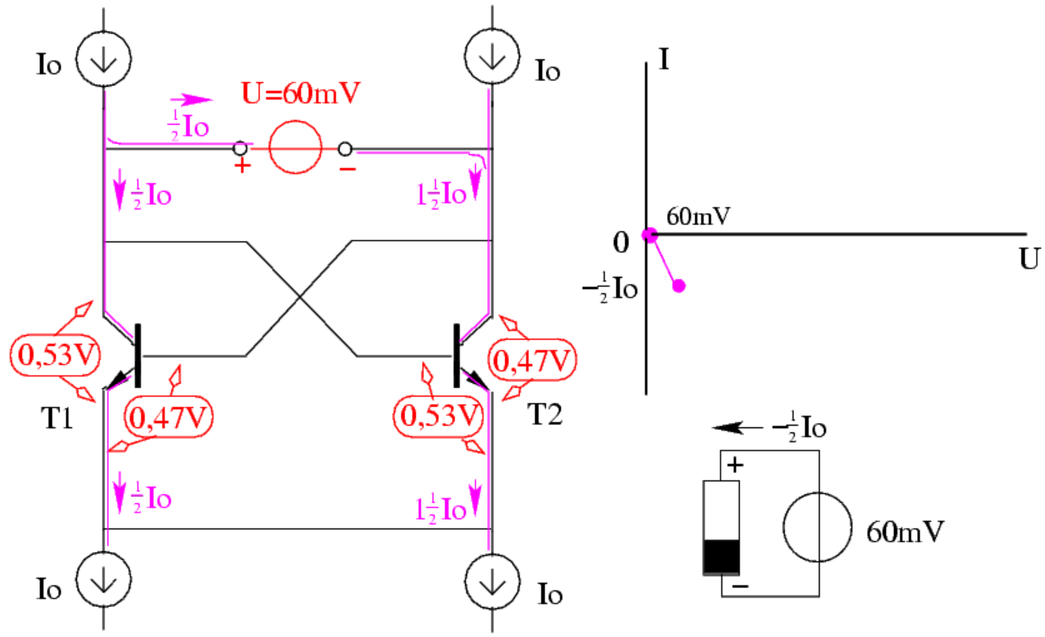
We start by analyzing the circuit without the coil connected. Instead we connect a voltage source on the two terminals. The initial value of this voltage source is 0V. As we can see the voltage source of 0V causes a balanced situation. Each I_0 flows through its "own" transistor. Because the collector of the each transistor is connected to the base of the other transistor, U_{ce} is equal to U_{be} of both transistors. So in this situation both transistors are just enough turned on to let I_0 ($\approx$1mA) flow through each transistor and because of that, U_{be} must be around 0.5V and thus is U_{ce} also around 0.5V. Since each Io flows through its "own" transistor, there will not flow any current through our connected voltage source.





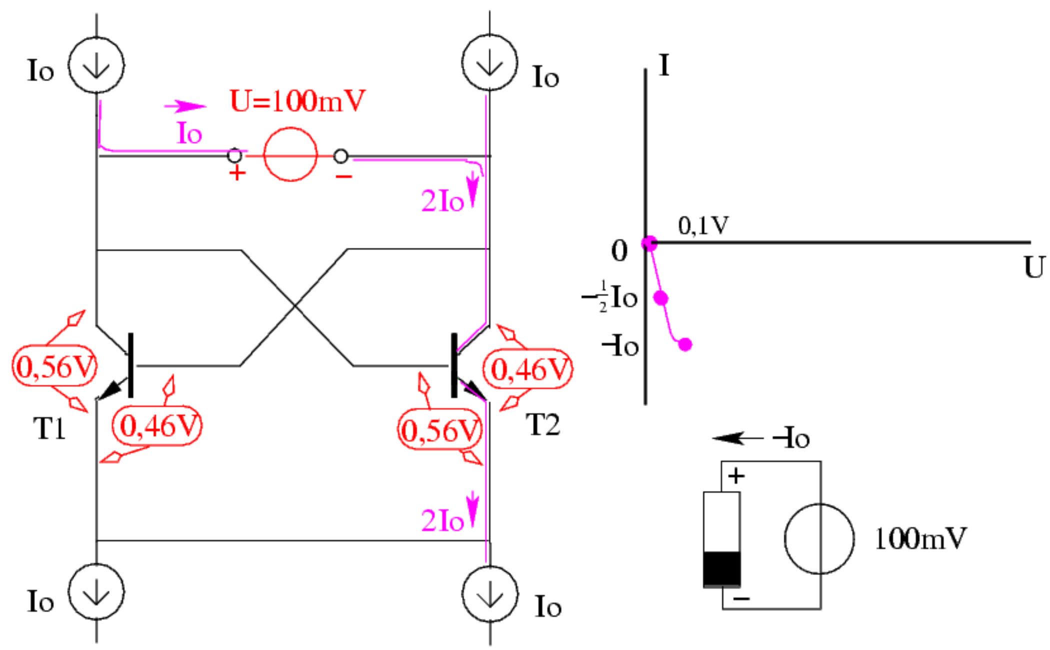


We now turn on the voltage source and set the voltage to 60mV. Due to this voltage the collector of T2 will pulled down and the collector of T1 will be pulled up. Because the base of one transistor is connected to the collector of the other, the base voltage of T1 will decrease, and the base voltage of T2 will increase. As a result, T1 turns more off and T2 turns more on. So the left I_0 current cannot flow entirely through T1, but starts to flow through T2, and therefore through our voltage source.

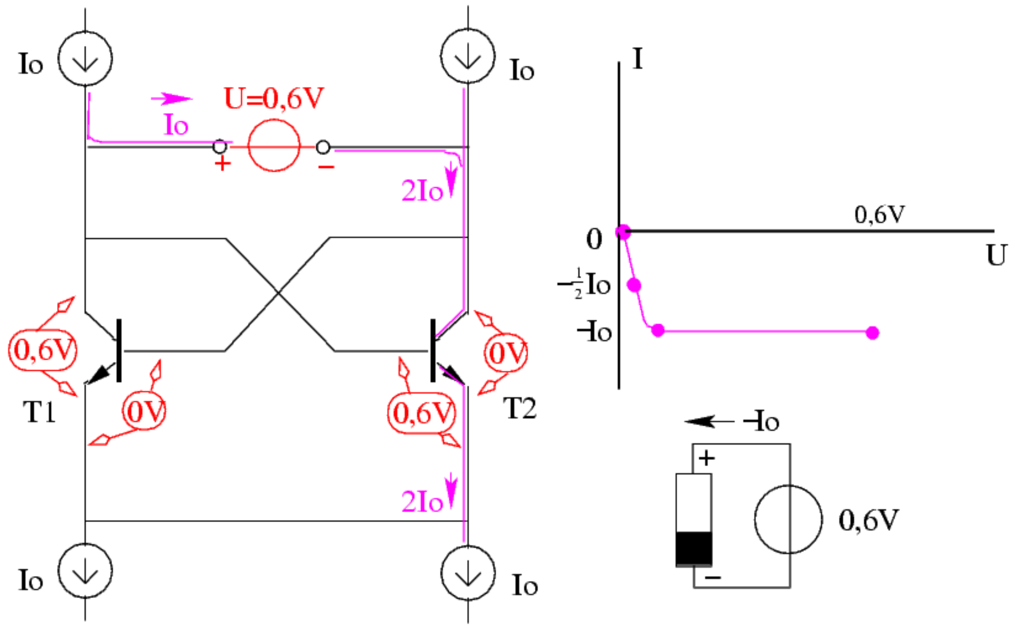


By seeing this we can conclude that if we turn a positive voltage on the terminals of our circuit, the circuit will deliver current and there will flow a negative current through our circuit. What we see is a non-linear resistor with negative resistance, because when the voltage is increasing the current is decreasing.

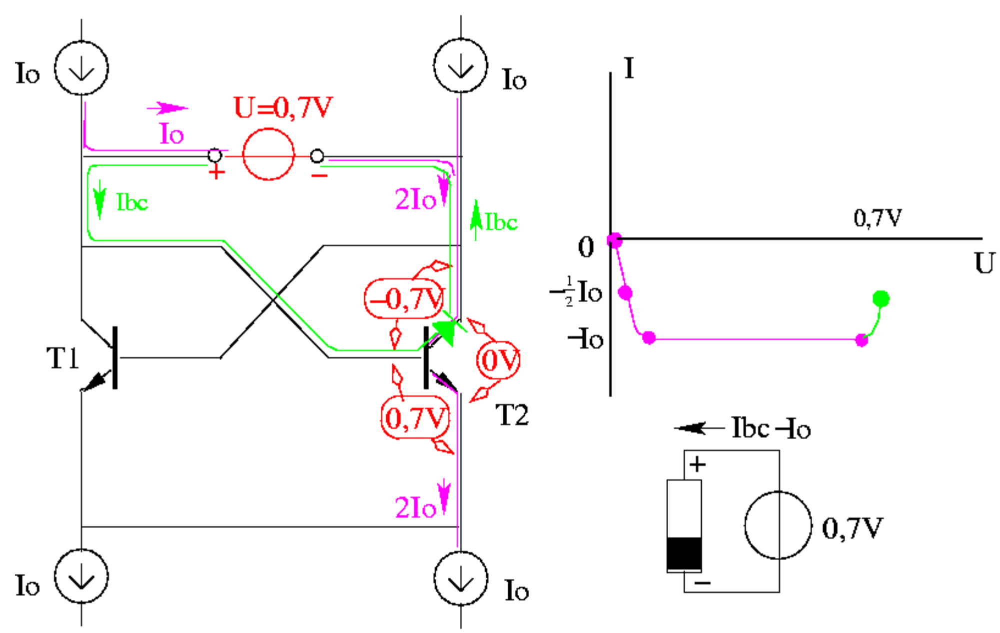
We now increase the voltage up to 100 mV. The T1 base voltage will decrease even more and the T2 base voltage will increase even more. This causes T1 to turn off more and T2 to turn on more. When a certain voltage is reached all current flows through T2 and nothing flows through T1. At this voltage I_0 flows through our voltage source.



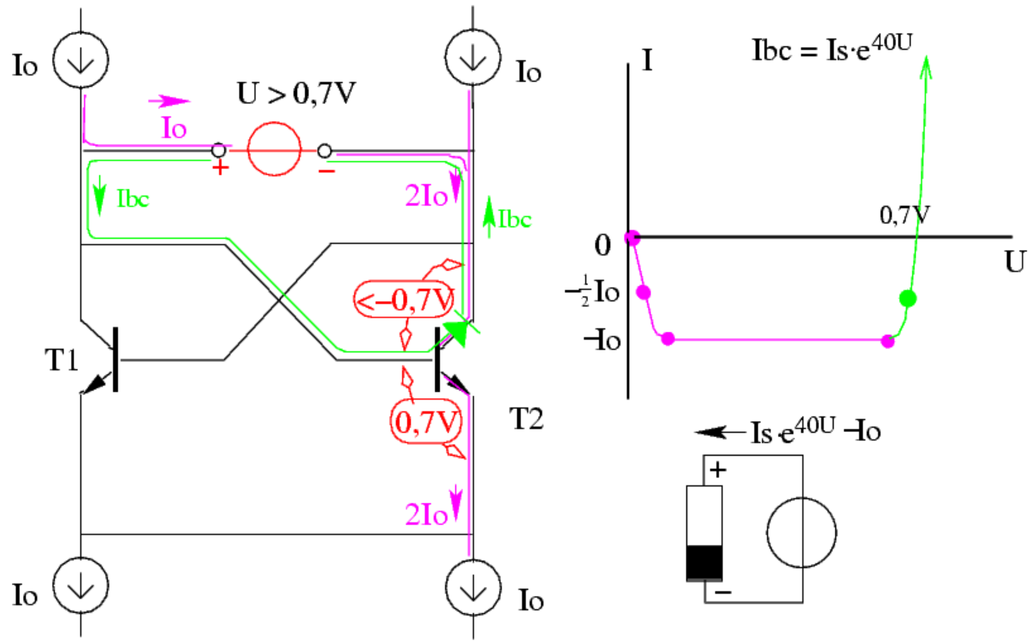
Increasing the voltage further does not increase the current through the voltage source. This can easily be understood since the upper left current source cannot deliver more than current than I_0. What we have now is an active resistor with a value of -∞ Ohm, with other words a current source. While increasing the source voltage more and more, U_{be} of T1 will decrease to 0, and U_{be} of T2 will increase to around 0.6V.



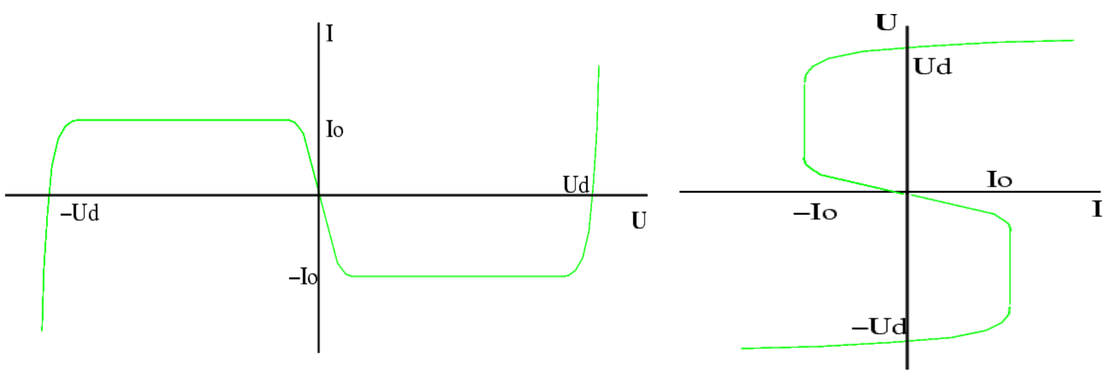
Can we increase the source voltage more and more while the current stays constant? The answer is no. T2 is a bipolar transistor and like every bipolar transistor it consists of two diodes, the *be* diode and the *bc* diode. The *bc* diode is directly connected to our variable voltage source, and as soon as the voltage becomes larger than 0.7V, the *bc* diode starts to conduct. This causes the current I_{bc} to flow from the voltage source through the *bc* diode. Our circuit now has the I-U relation $i = I_s e^{\wedge}(40u) - I_0$.



When the *bc* diode of T2 conducts more current than I_0, the circuit acts like a passive positive non-linear resistance.

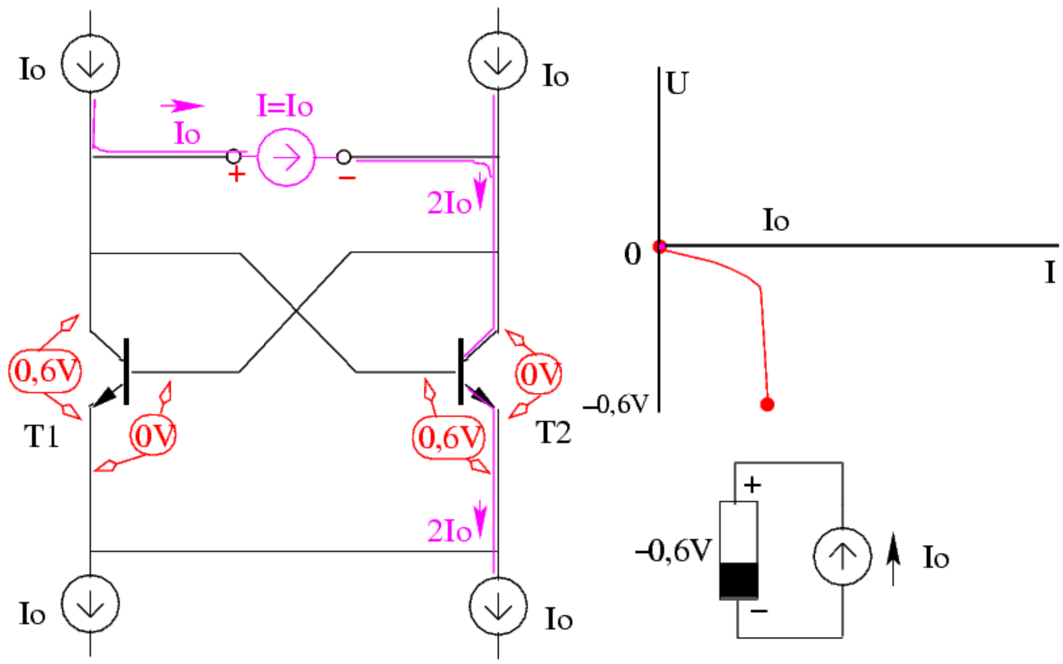


We have now found the positive part of the I-U plot. Since the circuit is symmetric the negative part can be derived in the same way.
If we connect a current source instead of a voltage source on the two terminals, we will get the same plot but with U as function of I. In the first and third quadrants of the plots the circuit is passive (consumes power), and in the second and third quadrants it is active (delivers power).

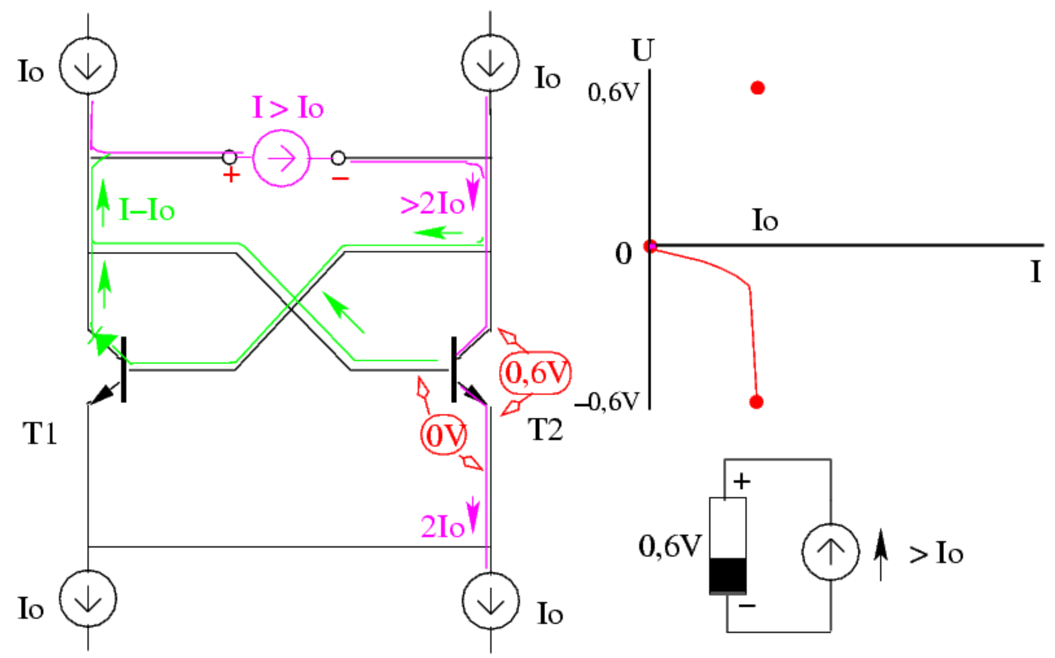


Note that if we drive the circuit with a current source, there are several voltages between - I_0 and + I_0 that are corresponding with the current. If we start with a negative current the voltage stays negative until is I_0, and if we start with a positive current the voltage will be positive until $-I_0$ is reached. So the voltage corresponding with a current between - I_0 and + I_0 is either positive or negative, depending on the situation before. This hysteresis effect proves the existing memory in our circuit.

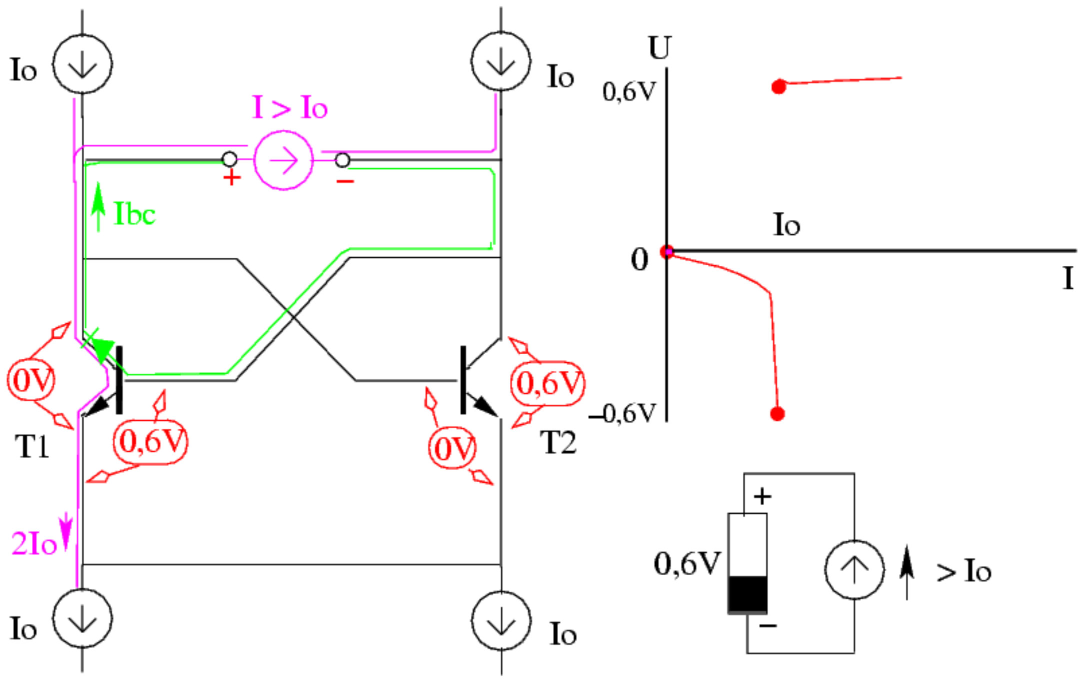
We also note that if we drive our circuit with a current source, the circuit will change voltage polarity as soon as the current source reaches I_0. How can we explain this?



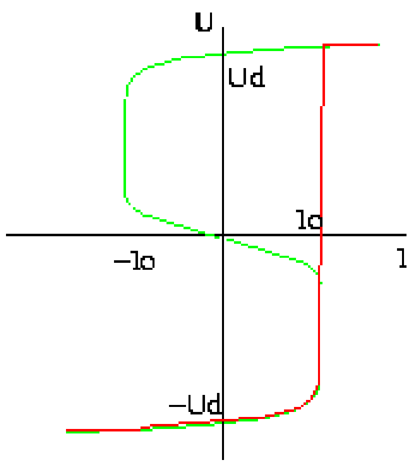
As we can see, the current source is exactly I_0. This causes the voltage over our current source to be -0.6V. Increasing the current source further we get.



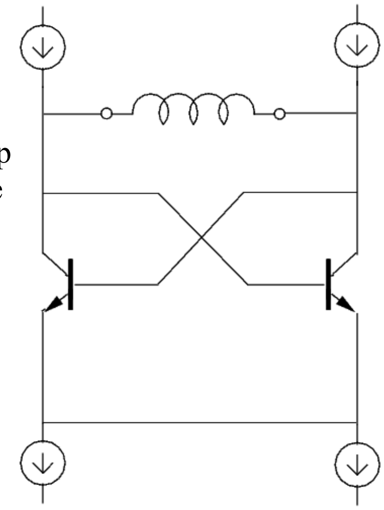
As earlier mentioned, the upper left current source cannot deliver more than more than I_0. The rest of the current for our source is coming from the collector of T1 and the base of T2. This small current from b of T2 will become zero or even negative, which causes T2 to turn off, and so immediately the U_{ce} of T2 will become high. This causes T1 to turn on. We can now see that the voltage over the current source has changed polarity.



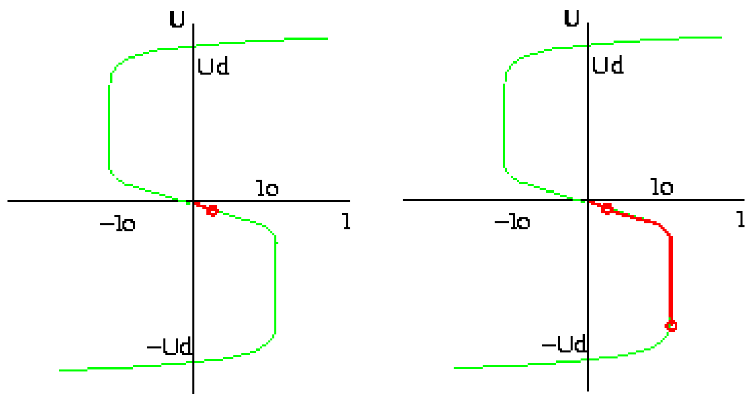
The path we have followed is shown below.



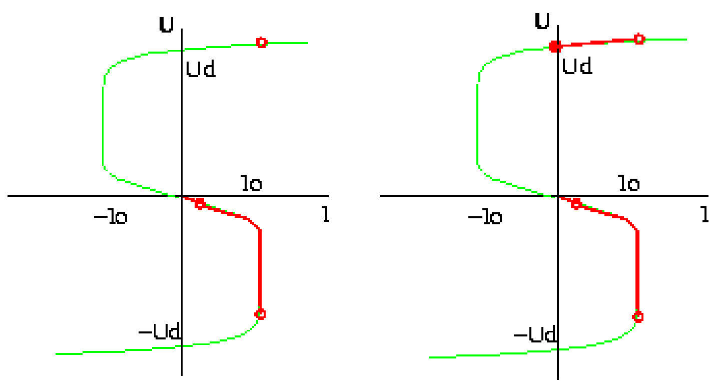
3.2 Connecting the coil



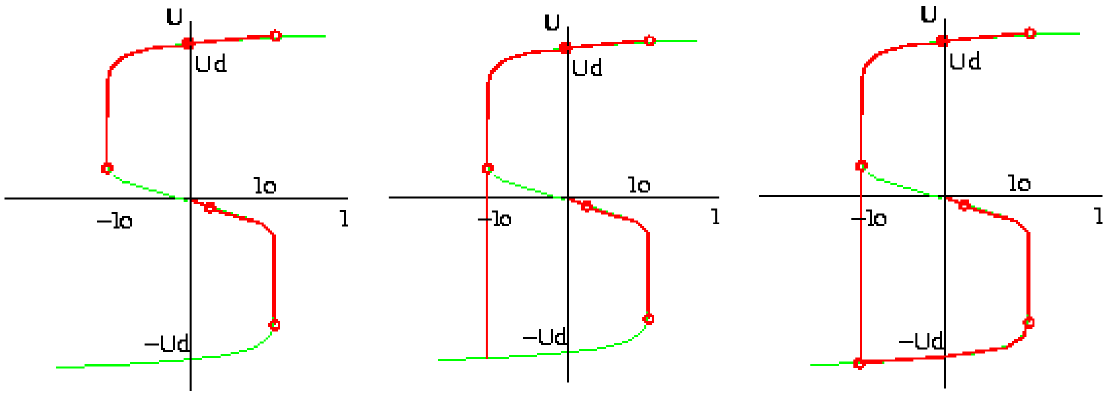
With the gathered information about U-I plots, we can understand what will happen if we connect a coil to our circuit.
Let us start from the origin point. Because of noise picked up by the coil, a small step current will start to flow through the coil. Let us assume that the initial noise signal decreases the current through the coil. If we define the coil voltage to be the same as the circuit voltage, we get that the current through the coil has the reverse polarity as the current through the circuit ($I_{coil} = -I_{circuit}$). This is why a decreasing current through the coil causes an increasing current in our circuit, as can be seen in the graphs below.



For a positive current, a negative voltage will be created by our circuit. This negative voltage will cause the current through the coil to decrease even more ($u_L = Ldi_L / dt$), and so we are moving in the graph. Since we are in part of the graph where the circuit is active, the coil will be charged. This procedure will go on until the current exceeds I_0. What then happens, as we have already explained before, is that the voltage of our circuit will change polarity. This can be seen in the right graph below.

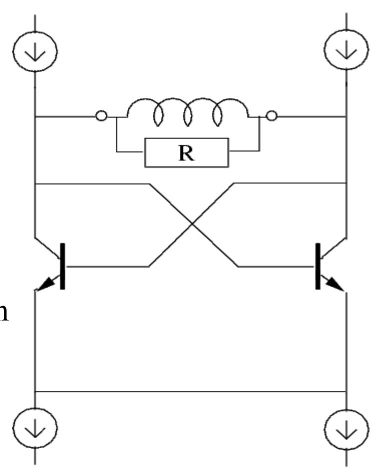


We are now in the passive state of our circuit and the circuit will thus consume energy from our coil. The coil will discharge and its current will increase (note that the absolute current through the coil will decrease), since the voltage now is positive. When we have reached 0A the coil is completely discharged and the circuit comes into its active state again. The positive voltage on the coil still causes the current through the coil to increase, and thus the current through the circuit to decrease ($dI_{coil}/dt = -dI_{circuit}/dt$). The coil is again charging and we are moving to the left in the graph. At a given moment the current through the coil will exceed $-I_0$ and our circuit will change voltage polarity again. With the same kind of reasoning we can complete the oscillation loop.



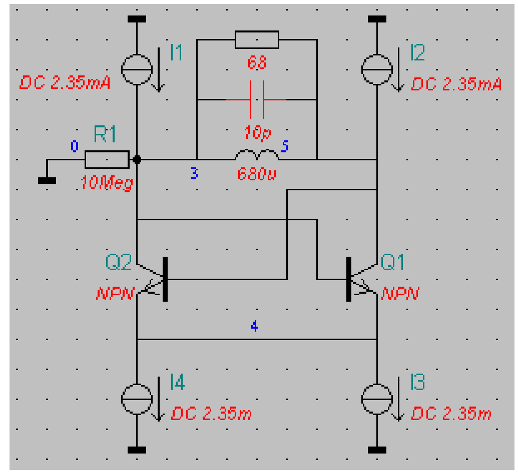
3.3 Adjusting the frequency of the oscillation

By choosing a larger I_0 we can decrease the frequency of our oscillator. This is because the coil then needs to reach a larger current before the circuit will change polarity. Since the threshold voltage U_d will almost stay the same (about 0.6-0.7V), a large I_0 results in more time needed to reach I_0.
How can we decrease the frequency of our oscillator, without increasing I_0? This is important, because a large I_0 results in larger power consumption. We have to find a way to slow down the charge-up and charge-down time of our coil.
We can understand that if we want to discharge a capacitor as slow as possible, we need to keep the connected resistance large. Looking at an infinitely small time step where the voltage is constant (voltage cannot change instantaneous over a capacitor), we realize that the large resistance minimizes the current and thus the power. To use a coil instead of a capacitor, we do the dual thing of connecting a small resistance. Once again looking at an infinitely small time step where the current is constant (current cannot change instantaneous in an inductor), we realize that the small resistance minimizes the voltage and thus the power. So we can conclude that if we want to discharge the coil less fast, we need to decrease the resistance connected to the coil. We can do that in our circuit by connecting a resistor parallel to the coil.
But as easily understood there is a minimal value of the resistor we can connect. Our coil still needs to be able to reach I_0, else our circuit will never change polarity and thus never oscillate.

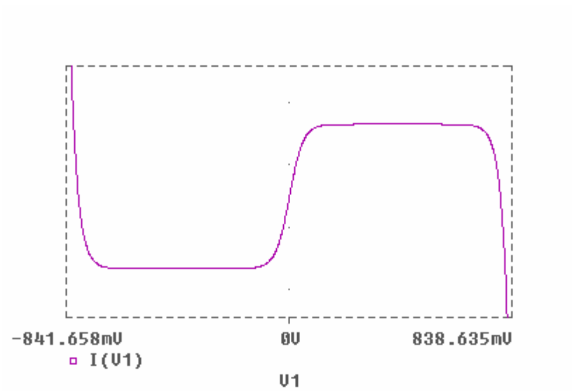


4. Simulating the circuit with SPICE

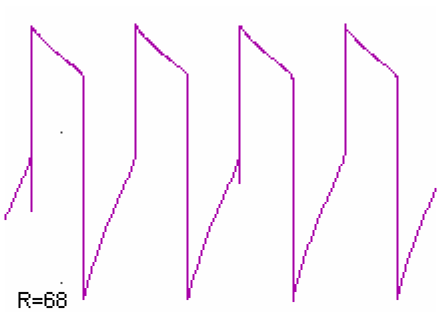
There is not too much to say about the SPICE analysis. It was pretty straight forward except for some things. For the simulation to work we had to put a small capacitor in parallel with the coil and an extra path to earth through a large resistance. To the right is the simulated circuit.

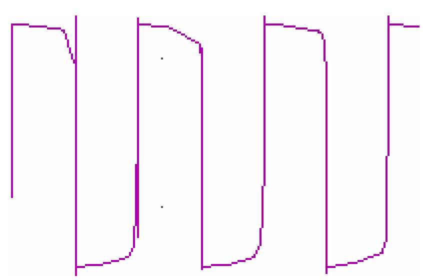


Below is an I-U plot of the two balanced transistor circuit.



We can see that the waveforms in SPICE are coherent with the waveforms that were measured.





5. Simulating the circuit with MATLAB

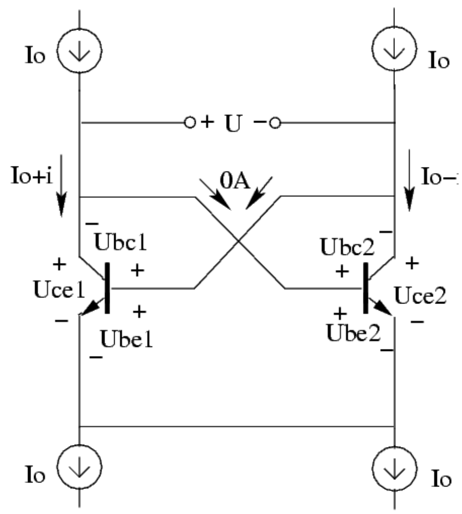
We here describe how the circuit can be analyzed analytically with MATLAB. We start by analyzing the circuit without the coil.

5.1 Analyzing the circuit without the coil

The transistor model we are using is:

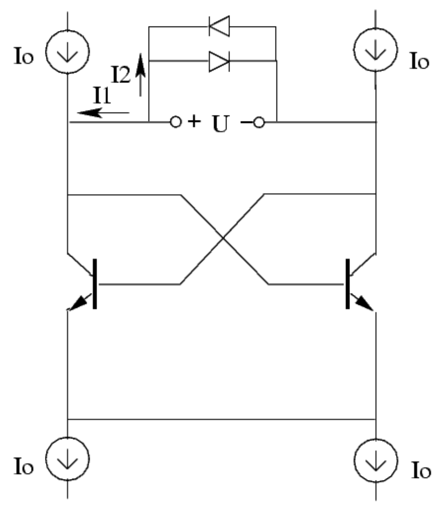
$u_{bc} > -V_t$:	$u_{bc} \le -V_t$:
$I_{be} = 0,$	*extra diode parallell on base – collector*
$I_{ce} = I_s e\wedge(40u_{be})$	$I_d = I_s e\wedge(40u_{be}) \quad I_{sd} = 10\wedge{-16}$

We now apply the transistor model on our circuit. First for $u_{bc} > -V_t$. The following the following equations can be extracted from the Kirchhoff laws.



$$I_0 + i = I_s e\wedge(40u_{be1})$$
$$I_0 - i = I_s e\wedge(40u_{be2})$$
$$u_{be2} = u_{be1} + u$$
$$\Rightarrow u = \frac{1}{40}\ln\left(\frac{(I_0 - i)}{(i - I_0)}\right)$$

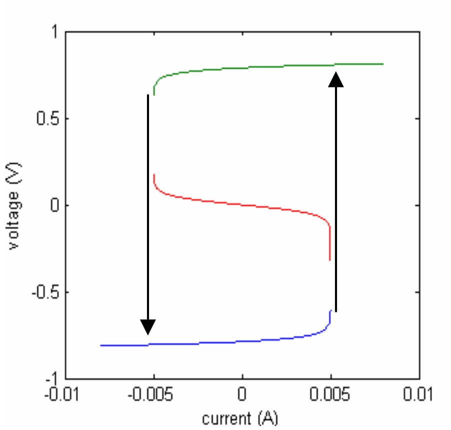
Now for $u_{bc} \le -V_t$. Once again applying the transistor model gives.



$$i = i_1 + i_2$$
$$u > V_t :$$
$$i_1 = -I_0$$
$$i_2 = I_{sd}\, e\wedge(40\, u)$$
$$\Rightarrow u = \frac{1}{40}\ln\left(\frac{(i + I_0)}{I_{sd}}\right)$$
$$u < -V_t :$$
$$i_1 = I_0$$
$$i_2 = -I_{sd}\, e\wedge(-40\, u)$$
$$\Rightarrow u = -\frac{1}{40}\ln\left(\frac{(I_0 - i)}{I_{sd}}\right)$$

Writing the three cases as one we get.

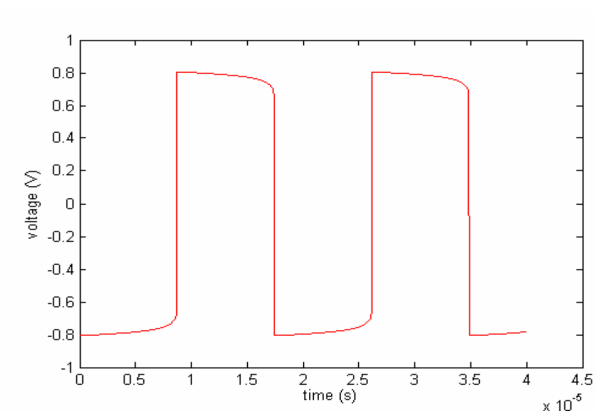
$$u = \begin{cases} u \le -V_t : & u = -\dfrac{1}{40}\ln\left(\dfrac{(I_0 - i)}{I_{sd}}\right) \\ |u| < V_t : & u = \dfrac{1}{40}\ln\left(\dfrac{(I_0 - i)}{(i - I_0)}\right) \\ u \ge V_t : & u = \dfrac{1}{40}\ln\left(\dfrac{(i + I_0)}{I_{sd}}\right) \end{cases}$$

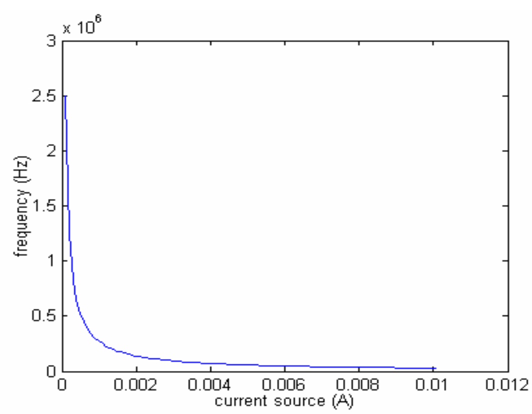


Note that the plot in reality is continuous. We now consider what happens when we connect the coil to this circuit. We then get, that the equation above and the inductor equation $V_L = LdI_L/dt$ together describes the behavior of the circuit. Combining the equations gives $u = -Ldi/dt$. With this equation and looking at the UI-plot we can understand how the oscillator works. Say that we start at the lower left part of the plot. The voltage is at this point negative which with $u = -Ldi/dt$ indirectly says that the current is increasing. We are thus moving to the right on the blue curve in the plot. When we reach 5mA (I_0), the blue line can be considered almost vertical. When we are moving along this vertical line the current is constantly increasing (since the voltage still is negative) and will eventually exceed I_0. When this happens, we will do a jump to the top right of the plot and the voltage will thus change polarity. Note that this jump in the plot does not interfere with the physical nature of an inductor where the current cannot change instantly. In the same way, we now move along the green curve until we hit $-I_0$ and there jump down back to the blue curve. The path around the plot that we have made is one oscillation, and it is the steady state that the circuit eventually will enter. The time it takes to move along this path is thus the period of the oscillation.

To calculate the frequency of the oscillation in MATLAB we first rewrite $u = -Ldi/dt$ as $di = -u\ dt/L$. We then use this equation to calculate the current for each time step. The voltage can then be calculated from the current. When we hit the I_0 or $-I_0$ we do the appropriate jump. The time it took to get back to the original position is stored to calculate the oscillation frequency.

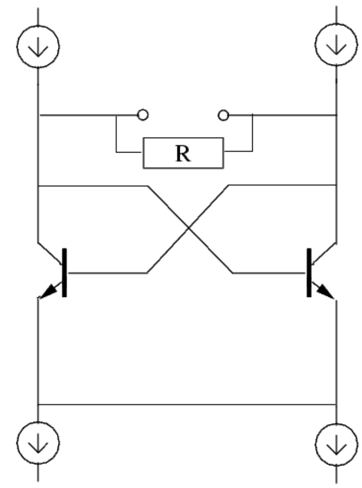
Here are two plots showing the oscillation and the effect of the current source on the frequency. We can clearly see that the frequency is inversely proportional to the current source.





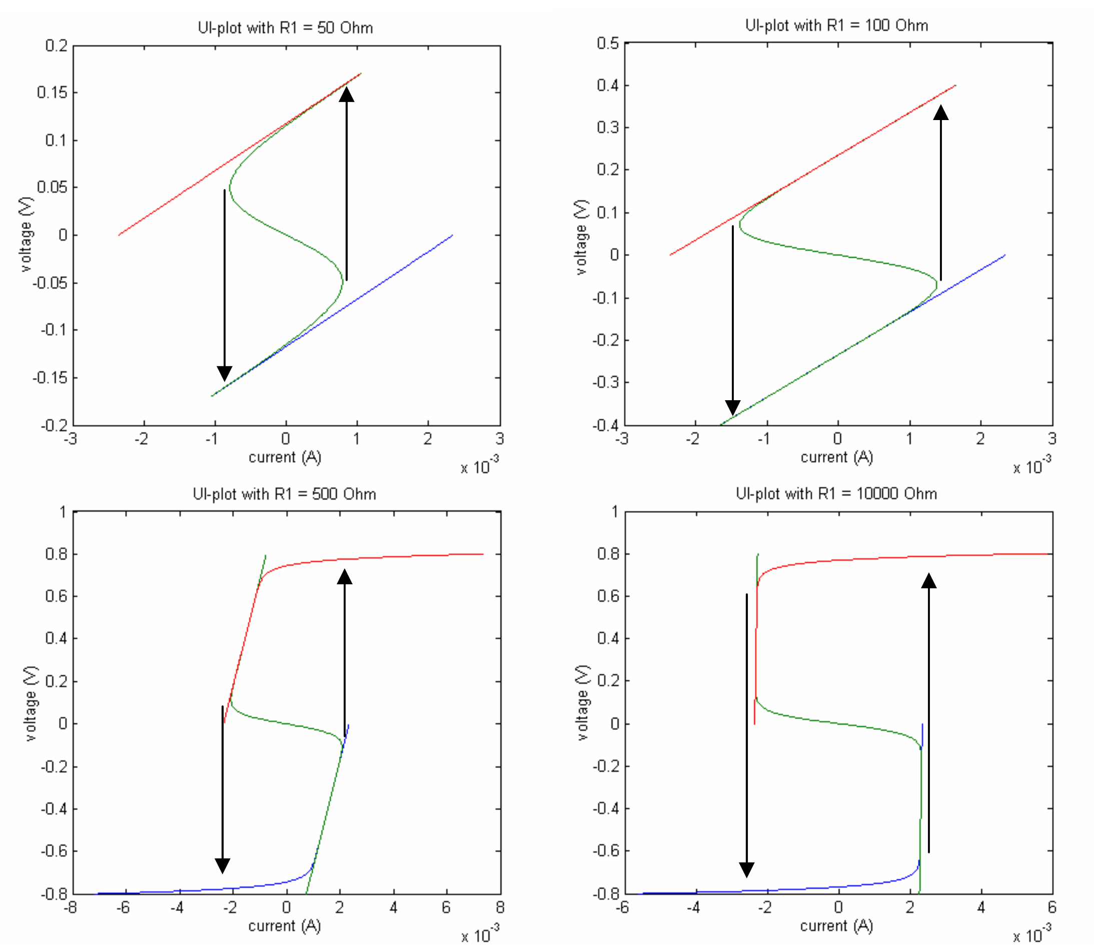
5.2 Adding a resistor in parallel

We now analyze the circuit with a resistance in parallel as shown in the picture to the right. For this we need to derive new equations for the U-I relation. Since the resistance is in parallel with the old circuit we can simply add up the currents of the individual U-I equations. We first rewrite the old *u=f(i)* equations in the form *i=f(u)* and then add up the resistance current *u/R*.



$$i = \begin{Bmatrix} I_0 - I_{sd} e\hat{}(-40u) + u/R \\ I_0 \dfrac{1 - e\hat{}(40u)}{1 + e\hat{}(40u)} + u/R \\ I_{sd} e\hat{}(40u) - I_0 + u/R \end{Bmatrix}$$

Below are U-I plots for different resistance values.



The curves that describe the oscillation depend on the resistance value. In the first plot the red and the blue curves are not important, but in the forth plot they are essential. To calculate the oscillation frequency in MATLAB we make the approximation that the oscillation path only follows the red and blue curves.
To use our existing model in MATLAB to calculate the frequency, we need to know when to do the jump (by I_0 does not work this time) and rewrite the functions above in the form *u=f(i)*. We can calculate the "jump current" by moving from the origin, up along the green curve, saving the minimal current reached. The saved current then represents the current where the green curve "turns" (see plot). Rewriting the functions in the form *u=f(i)* can be done by using the *lambert*-function, also called the *W*-function. It is defined as:

$$y = xe\hat{}\,x$$
$$W(y) = x$$

Here is a rewrite of the equation that represents the blue curve in the form *u=f(i)*. The equation for the red curve can similarly be derived.

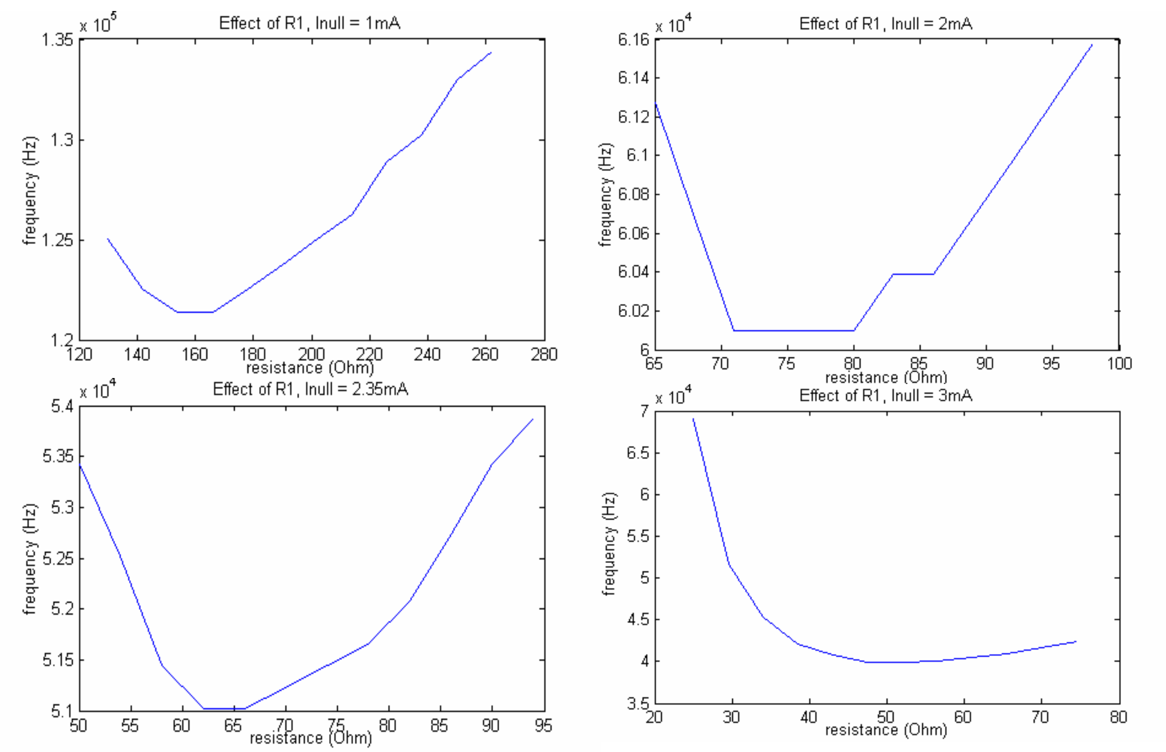
$$i = I_0 - I_{sd} e\hat{}(-40u) + u/R$$
$$(i - I_0)(-40R) = 40RI_{sd} e\hat{}(-40u) - 40u$$
$$e\hat{}((i - I_0)(-40R)) = e\hat{}(40RI_{sd} e\hat{}(-40u) - 40u)$$
$$e\hat{}((i - I_0)(-40R)) = e\hat{}(-40u)e\hat{}(40RI_{sd} e\hat{}(-40u))$$
$$40RI_{sd} e\hat{}((i - I_0)(-40R)) = 40RI_{sd} e\hat{}(-40u)e\hat{}(40RI_{sd} e\hat{}(-40u))$$
$$W(40RI_{sd} e\hat{}((i - I_0)(-40R))) = 40RI_{sd} e\hat{}(-40u)$$
$$u = -1/40 \ln(1/(40RI_{sd})W(40RI_{sd} e\hat{}((i - I_0)(-40R)))$$

The equations that are necessary for the frequency calculation in MATLAB are thus.

$$u = 1/40 \ln(1/(40RI_{sd})W(40RI_{sd} e\hat{}((i + I_0)(40R))) \qquad \text{Path to follow}$$
$$i = I_0 \frac{1 - e\hat{}(40u)}{1 + e\hat{}(40u)} + u/R \qquad \text{Used to determine when to jump}$$
$$u = -1/40 \ln(1/(40RI_{sd})W(40RI_{sd} e\hat{}((i - I_0)(-40R))) \qquad \text{Path to follow}$$

The same basic algorithm, as in the case with no resistance, is used to calculate the frequency.

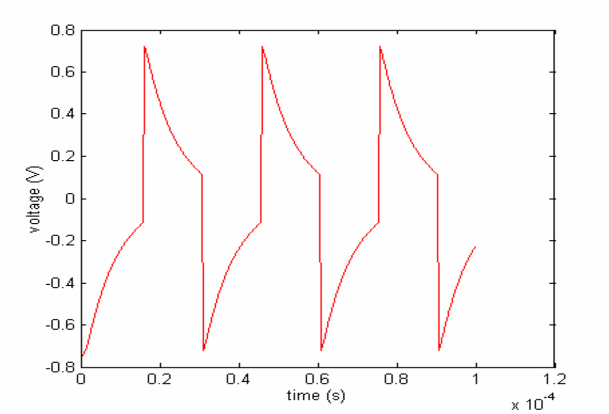
Here are plots showing the effect of the resistance on the frequency, for different current sources.



We can see that each current source has a corresponding minimum frequency resistance.

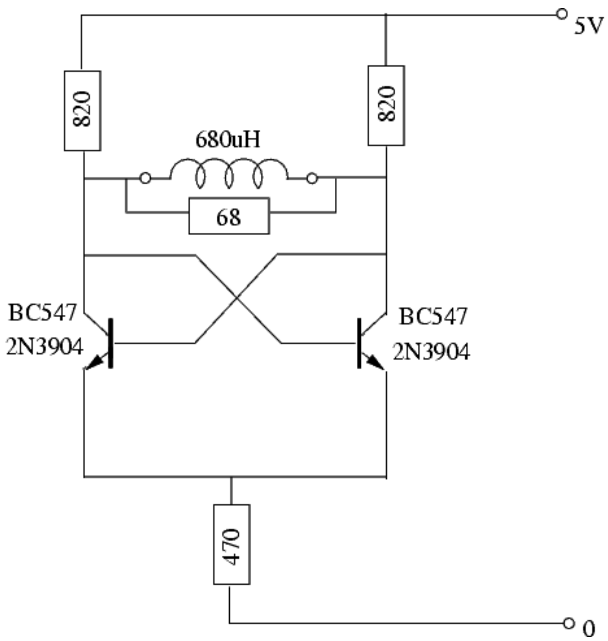
The third plot represents the final circuit and we can see that the minimum frequency is about 50 kHz with a corresponding resistance of about 65 Ohm. The resistance used in the built circuit was 67 Ohm, which tells us that the simulation is correct and fairly accurate.

Below is an oscillation plot for the final circuit.



6. The final circuit

Since one of the requirements was that the circuit had to consume minimal power, we chose to use the parallel resistance to decrease the necessary current sources. The disadvantage of this is that the waveform looks less like a square waveform.
Below is a schematic of the built circuit. This circuit oscillates at 50 kHz and consumes 24 mW of power.



7. MATLAB scripts

```
% print the UI plot of the circuit with no R
function main
% circuit params
L = 0.68*10^-3; % 0.68 mH
Isd = 1*10^-16; % 10 fA
Inull = 5*10^-3; % 5 mA

% model params
n = 1000;
StartI = -Inull - 0.003;
EndI = Inull - 0.00001;
DeltaI = (EndI - StartI)/n;
Current1(1) = StartI;
Voltage1(1) = (-1/40) * (log(Inull - Current1(1)) - log(Isd));
for j=1: n+1;
  Current1(j+1) = Current1(j) + DeltaI;
  Voltage1(j+1) = (-1/40) * (log(Inull - Current1(j+1)) - log(Isd));
end;


StartI = -Inull + 0.00001;
EndI = Inull + 0.003;
DeltaI = (EndI - StartI)/n;
Current2(1) = StartI;
Voltage2(1) = (1/40) * (log(Inull + Current2(1)) - log(Isd));
for j=1: n+1;
Current2(j+1) = Current2(j) + DeltaI;
  Voltage2(j+1) = (1/40) * (log(Inull + Current2(j+1)) - log(Isd));
end;

StartI = -Inull + 0.00001;
EndI = Inull - 0.00001;
DeltaI = (EndI - StartI)/n;
Current3(1) = StartI;
Voltage3(1) = (1/40) * (log(Inull - Current3(1)) - log(Current3(1) + Inull));
for j=1: n+1;
    Current3(j+1) = Current3(j) + DeltaI;
    Voltage3(j+1) = (1/40) * (log(Inull - Current3(j+1)) - log(Current3(j+1) + Inull));
end;

% print diagram
plot(Current1, Voltage1, Current2, Voltage2, Current3, Voltage3);
xlabel('current (A)');
ylabel('voltage (V)');



% print oscillation of the circuit with no R
function main
% circuit params
L = 0.68*10^-3; % 0.68 mH
Isd = 1*10^-16; % 10 fA
Inull = 5*10^-3; % 5 mA

% model params
T = 0.04*10^-3;
n = 1000;
DeltaTime = T/n;

% start values
i(1) = -Inull;
u(1) = (-1/40) * (log(Inull - i(1)) - log(Isd));

%time calculation
Time(1) = 0;
for j=2: n+2;
```

```
  Time(j) = Time(j-1) + DeltaTime;
end;

for j=1: n+1;
  % calculate the new current from the old current and the old voltage
  i(j+1) = i(j) - DeltaTime * u(j) / L;

  % top half of plot?
  if(u(j) > 0)
    if(i(j+1) <= -Inull) % jump to bottom half of the plot?
      u(j+1) = (-1/40) * (log(Inull - i(j+1)) - log(Isd));
    else
      u(j+1) = (1/40) * (log(Inull + i(j+1)) - log(Isd));
    end;
  % bottom half of the plot
  else
    if(i(j+1) >= Inull) % jump to top half of the plot?
      u(j+1) = (1/40) * (log(Inull + i(j+1)) - log(Isd));
    else
      u(j+1) = (-1/40) * (log(Inull - i(j+1)) - log(Isd));
    end;
  end;
end;

% print diagram
plot(Time,u,'-r')
xlabel('time (s)');
ylabel('voltage (V)');

%plot(Time,i,'-r')
%xlabel('time (s)');
%ylabel('current (A)');

%plot(i,u)
%xlabel('current (A)');
%ylabel('voltage (V)');




% print the frequency as a function of current for the circuit with no R
function main
% model params
StartI = 0.0001;
EndI = 0.01;
n = 100;
DeltaI = (EndI - StartI) / n;
Current(1) = StartI;
Freq(1) = CalcFreq(StartI);

% calculate the frequency for each current
for j=1: n+1;
  Current(j+1) = Current(j) + DeltaI;
  Freq(j+1) = CalcFreq(Current(j+1));
end;

% print diagram
plot(Current, Freq);
xlabel('current source (A)');
ylabel('frequency (Hz)');




% calculate the oscillation frequency for the specified current source for
% the circuit with no R
function Freq = CalcFreq(Inull)

% circuit params
L = 0.68*10^-3; % 0.68 mH
Isd = 1*10^-16; % 10 fA
```

```
% model params
T = 0.04*10^-3;
n = 1000;
DeltaTime = T/n;
Time = 0;

% calculate how long it takes to go through the bottom half of the plot
i(1) = -Inull;
u(1) = (-1/40) * (log(Inull - i(1)) - log(Isd));

for j=1: 1000000000;
  % calculate the new current
  i(j+1) = i(j) - DeltaTime * u(j) / L;
  Time = Time + DeltaTime;

  % hit Inull?
  if(i(j+1) >= Inull)
    break;
  end;

  % calculate the new voltage
  u(j+1) = (-1/40) * (log(Inull - i(j+1)) - log(Isd));
end;

% calculate how long it takes to go through the top half of the plot
i(1) = Inull;
u(1) = (1/40) * (log(Inull + i(1)) - log(Isd));

for j=1: 1000000000;
  % calculate the new current
  i(j+1) = i(j) - DeltaTime * u(j) / L;
  Time = Time + DeltaTime;

  % hit Inull?
  if(i(j+1) <= -Inull)
    break;
  end;

  % calculate the new voltage
  u(j+1) = (1/40) * (log(Inull + i(j+1)) - log(Isd));
end;

Freq = 1 / Time;




% calculate the frequency for a specified current source and resistance
function Freq = CalcFreq(Inull, R)

% circuit params
L = 0.68*10^-3; % 0.68 mH
Isd = 1*10^-16; % 10 fA

% model params
T = 0.04*10^-3;
n = 1000;
DeltaTime = T/n;
Time = 0;

% calculate the start and stop limits of the current
CurrentSpan = CalcCurrentSpan(Isd, Inull, R);

% calculate how long it takes to go through the bottom half of the plot
i(1) = -CurrentSpan/2;
u(1) = (-1/40) * (log(Inull - i(1)) - log(Isd));

for j=1: 1000000000;
  % calculate the new current
  i(j+1) = i(j) - DeltaTime * u(j) / L;
```

```
  Time = Time + DeltaTime;

  % hit span limit?
  if(i(j+1) >= CurrentSpan/2)
    break;
  end;

  % calculate the new voltage
  u(j+1) = CalcVoltageBottom(i(j+1), Isd, Inull, R);
end;

% calculate how long it takes to go through the top half of the plot
i(1) = CurrentSpan/2;
u(1) = (1/40) * (log(Inull + i(1)) - log(Isd));

for j=1: 1000000000;
  % calculate the new current
  i(j+1) = i(j) - DeltaTime * u(j) / L;
  Time = Time + DeltaTime;

  % hit span limit?
  if(i(j+1) <= -CurrentSpan/2)
    break;
  end;

  % calculate the new voltage
  u(j+1) = CalcVoltageTop(i(j+1), Isd, Inull, R);
end;

% return the frequency
Freq = 1 / Time;

% calculate the voltage from the current for bottom part of plot
function v = CalcVoltageBottom(i, Isd, Inull, R)
  v = (-1/40)*log(1/(40*R*Isd)*lambertw(exp((i-Inull)*(-40*R))*40*R*Isd));

% calculate the voltage from the current for top part of plot
function v = CalcVoltageTop(i, Isd, Inull, R)
  v = (1/40)*log(1/(40*R*Isd)*lambertw(exp((i+Inull)*40*R)*40*R*Isd));

% calculate the current from the voltage for center part of plot
function i = CalcCurrentCenter(v, Isd, Inull, R)
  i = Inull*((1-exp(40*v))/(1+exp(40*v)))+v/R;

% calculate the current span so we know when to stop
function CurrentSpan = CalcCurrentSpan(Isd, Inull, R)
VStart = 0;
VStop = 0.8;
n = 100;
DeltaV = (VStop-VStart)/n;
v(1) = VStart;
i(1) = CalcCurrentCenter(v(1), Isd, Inull, R);
mini = i(1);

for j=1: n+1;
  v(j+1) = v(j) + DeltaV;
  i(j+1) = CalcCurrentCenter(v(j+1), Isd, Inull, R);

  if(i(j+1) < mini)
    mini = i(j+1);
  end;
end;

% return the span
CurrentSpan = 2 * -mini;
```

```
% print the frequency as a function of the current source and a specified R
function main
% circuit params
R = 86;

% model params
StartI = 0.001;
EndI = 0.005;
n = 5;
DeltaI = (EndI - StartI) / n;
Current(1) = StartI;
Freq(1) = CalcFreqR(Current(1), R);

% calculate the frequency for each current
for j=1: n+1;
  Current(j+1) = Current(j) + DeltaI;
  Freq(j+1) = CalcFreqR(Current(j+1), R);
end;

% print diagram
plot(Current, Freq);
xlabel('current source');
ylabel('frequency');
```

```
% print the frequency as a function of R
function main
% circuit params
Inull = 0.00235;

% model params
StartR = 50;
EndR = 90;
n = 10;
DeltaR = (EndR - StartR) / n;
R(1) = StartR;
Freq(1) = CalcFreqR(Inull, R(1));

% calculate the frequency for each current
for j=1: n+1;
  R(j+1) = R(j) + DeltaR;
  Freq(j+1) = CalcFreqR(Inull, R(j+1));
end;

% print diagram
plot(R, Freq);
xlabel('resistance');
ylabel('frequency');
```

```
% print oscillation for a specified value of R
function main
% circuit params
R = 86; % 86 Ohm
L = 0.68*10^-3; % 0.68 mH
Isd = 1*10^-16; % 10 fA
Inull = 5*10^-3; % 5 mA

% model params
T = 0.0001;
n = 1000;
DeltaTime = T/n;

% start values
i(1) = -Inull;
u(1) = CalcVoltageBottom(i(1), Isd, Inull , R);
```

```
%time calculation
Time(1) = 0;
for j=2: n+2;
  Time(j) = Time(j-1) + DeltaTime;
end;

% calculate current span
CurrentSpan = CalcCurrentSpan(Isd, Inull, R);

for j=1: n+1;
  % calculate the new current from the old current and the old voltage
  i(j+1) = i(j) - DeltaTime * u(j) / L;

  % top half of plot?
  if(u(j) > 0)
    if(i(j+1) <= -CurrentSpan/2) % jump to bottom half of the plot?
      u(j+1) = CalcVoltageBottom(i(j+1), Isd, Inull, R);
    else
      u(j+1) = CalcVoltageTop(i(j+1), Isd, Inull, R);
    end;
  % bottom half of the plot
  else
    if(i(j+1) >= CurrentSpan/2) % jump to top half of the plot?
      u(j+1) = CalcVoltageTop(i(j+1), Isd, Inull, R);
    else
      u(j+1) = CalcVoltageBottom(i(j+1), Isd, Inull, R);
    end;
  end;
end;

% print diagram
plot(Time,u,'-r')
xlabel('time (s)');
ylabel('voltage (V)');

%plot(Time,i,'-r')
%xlabel('time (s)');
%ylabel('current (A)');

%plot(i,u)
%xlabel('current (A)');
%ylabel('voltage (V)');

% calculate the voltage from the current for bottom part of plot
function v = CalcVoltageBottom(i, Isd, Inull, R)
  v = (-1/40)*log(1/(40*R*Isd)*lambertw(exp((i-Inull)*(-40*R))*40*R*Isd));

% calculate the voltage from the current for top part of plot
function v = CalcVoltageTop(i, Isd, Inull, R)
  v = (1/40)*log(1/(40*R*Isd)*lambertw(exp((i+Inull)*40*R)*40*R*Isd));

% calculate the current from the voltage for center part of plot
function i = CalcCurrentCenter(v, Isd, Inull, R)
  i = Inull*((1-exp(40*v))/(1+exp(40*v)))+v/R;

% calculate the current span so we know when to stop
function CurrentSpan = CalcCurrentSpan(Isd, Inull, R)
VStart = 0;
VStop = 0.8;
n = 100;
DeltaV = (VStop-VStart)/n;
v(1) = VStart;
i(1) = CalcCurrentCenter(v(1), Isd, Inull, R);
mini = i(1);

for j=1: n+1;
  v(j+1) = v(j) + DeltaV;
  i(j+1) = CalcCurrentCenter(v(j+1), Isd, Inull, R);
  if(i(j+1) < mini)
    mini = i(j+1);
  end;
```

```
end;

% return the span
CurrentSpan = 2 * -mini;



% print the UI plot with a specified R
function Main
% circuit params
R = 86; % 86 Ohm
L = 0.68*10^-3; % 0.68 mH
Isd = 1*10^-16; % 10 fA
Inull = 2.35*10^-3; % 2.35 mA

% model params
VMin = -0.3;
VMax = -VMin;

% calculate the voltage for the left part of the plot
VStart = VMin;
VStop = 0.0;
n = 1000;
DeltaV = (VStop-VStart)/n;
Voltage1(1) = VStart;
Current1(1) = CalcCurrentLeft(Voltage1(1), Isd, Inull, R);

for j=1: n+1;
  Voltage1(j+1) = Voltage1(j) + DeltaV;
  Current1(j+1) = CalcCurrentLeft(Voltage1(j+1), Isd, Inull, R);
end;

% calculate the voltage for the center part of the plot
VStart = VMin;
VStop = VMax;
n = 1000;
DeltaV = (VStop-VStart)/n;
Voltage2(1) = VStart;
Current2(1) = CalcCurrentCenter(Voltage2(1), Isd, Inull, R);

for j=1: n+1;
  Voltage2(j+1) = Voltage2(j) + DeltaV;
  Current2(j+1) = CalcCurrentCenter(Voltage2(j+1), Isd, Inull, R);
end;

% calculate the voltage for the right part of the plot
VStart = 0.0;
VStop = VMax;
n = 1000;
DeltaV = (VStop-VStart)/n;
Voltage3(1) = VStart;
Current3(1) = CalcCurrentRight(Voltage3(1), Isd, Inull, R);

for j=1: n+1;
  Voltage3(j+1) = Voltage3(j) + DeltaV;
  Current3(j+1) = CalcCurrentRight(Voltage3(j+1), Isd, Inull, R);
end;

% print diagram
plot(Current1, Voltage1, Current2, Voltage2, Current3, Voltage3);
xlabel('current');
ylabel('voltage');

function i = CalcCurrentLeft(v, Isd, Inull, R)
  i = Inull - Isd*exp(-40*v)+v/R;

function i = CalcCurrentCenter(v, Isd, Inull, R)
  i = Inull*((1-exp(40*v))/(1+exp(40*v)))+v/R;

function i = CalcCurrentRight(v, Isd, Inull, R)
  i = -Inull + Isd*exp(40*v)+v/R;
```